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Assessing the statistical significance of palaeostress estimates: simulations using random fault-slips

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Abstract

Fault-slip analysis assumes that measured slip lineations on faults represent the direction of maximum resolved stress produced by single homogenous state of stress. To devise criteria for recognising natural data that do not comply with this assumption, the performance of fault-slip methods is examined when used to analyse unsuitable data; namely, faults and slip lineations with randomly chosen orientations.

Data quality is often judged by examining the average discrepancy between the orientation of actual slip lineation on each fault and the lineation theoretically predicted from the best-fit tensor. In this work, however, it is found that random faults also yield small angular misfits in conditions where eight or less faults are used. This criterion is therefore only useful for large samples of faults. Another test of data quality is to use the existence of tensors that are compatible with a given data set. However, even for random data, tensors can be found that are capable of explaining the lineation orientations. For example, the existence of compatible stress orientations deduced from the right dihedra method is no proof that the data meet the assumptions of the method. The probability of finding such tensors depends on the tolerance used when assessing fit, and the total number of trial tensors used. A more useful check on data quality is the proportion of trial tensors that fit data sets. For random data this proportion is found to decrease rapidly with sample size. For sample sizes greater than five faults, the expected proportion of tensors fitting is very small ($<1\%$). Statistical tests are proposed. This study emphasises the dangers of palaeostress determinations from small numbers of faults. All of the tests of quality increase in power as the number of faults in the sample increases. It is concluded that stress estimates based on eight or less faults should be treated with grave suspicion.

Keywords: Fault slip analysis; Statistics; Dynamics; Palaeostress

1. Introduction

Fault-slip analysis, sometimes referred to as palaeostress analysis, uses information from fault kinematics to infer palaeostresses. It involves the estimation of the nature of ancient stresses by the inversion of data consisting of the orientations of fault planes and of their associated slip lineations. The different methods available for fault-slip analysis are reviewed by Angelier (1994) and Ramsay and Lisle (2000, pp. 775–810). The vast majority of these methods are based on the following assumptions:

(1) The slip lineations are indicators of the direction of resolved shear stress at the time of fault movement.
(2) The direction of slip on a given fault is not influenced by interaction with other faults.
(3) The data are collected from faults that slipped under the influence of a common stress tensor.

A further assumption implicit in using fault displacements as data for estimating stresses is that the stress and strain tensors show a direct relationship. These assumptions are far-reaching and have provoked some to question the validity of fault-slip methods for stress estimation (e.g. Marrett and Allmendinger, 1990; Pollard et al., 1993; Twiss and Unruh, 1998; Roberts and Ganas, 2000). This short contribution concerns the third assumption above and considers the consequences of using data that do not comply with it.

Since the fulfilment of this assumption is crucial for the success of the stress inversion, the principal aim of this study is to devise criteria for the recognition of situations where the data do not meet the requirements of assumption 3. The ability to fend off the critics of palaeostress analysis will depend on the development of methods for testing the underlying assumptions. Our approach here is to explore, using computer

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simulation and artificial data, the performance of inversion methods under problematic conditions of poor data quality.

2. Heterogeneous fault-slip data

Stress analysis from geological structures is fraught with difficulties arising from the fact that stress fields change with time. Where data from an assemblage of structures are required, as is the case with fault-slip analysis, there is a real danger that the collected data may have been taken from structures that are not synchronous and therefore do not constitute a homogeneous data set, i.e. they are kinematically incompatible with a single stress tensor. In such circumstances, the stress tensor derived from inverting these data would be of doubtful meaning.

In this study we investigate, using existing inversion methods, the results obtained from the analysis of heterogeneous fault-slip data sets. These data sets have been generated in order to mimic the samples collected from a region with a complex stress history, where the lineations have been produced by the action of a wide variety of stress tensors. These are generated by random selection of fault plane orientation and lineation pitch. They therefore represent the worst case scenario in terms of the quality of natural data.

3. The right dihedra method

The right dihedra method (Pegararo, 1972; Angelier and Mechler, 1977) is a geometrical method for constraining the feasible orientations of the principal stress axes from fault-slip data. As shown by McKenzie (1969), a single fault with a known slip orientation and a known sense of slip limits the possible orientation of the σ_1 and σ_3 to separate fields on the sphere. These fields are bounded by two orthogonal planes, the fault plane and the plane perpendicular to the direction of slip, that together form a right dihedron. These fields are usually displayed in stereographic projection. Combining the constraints from several faults recorded from a single station leads to the definition of σ_1 and σ_3 orientation fields that are compatible with different percentages of the faults (Fig. 1).

If the slips on a set of faults all result from a single stress tensor, the stereogram should display σ_1 and σ_3 orientation regions compatible with 100% of the faults. For this reason the existence of 100% regions is sometimes used as an indicator of data quality. In fact Carey-Gailhardis et al. (1992) have suggested that the existence of 100% regions is an essential requirement but not a sufficient requirement for determining stresses from this method. In addition, the presence of 100% regions occupying small areas on the stereogram gives the impression that the solution is tightly constrained by the data.

In this paper we evaluate the use of the 100% σ_1 region as a data quality indicator. To gain a better understanding of the significance of the 100% σ_1 area we have constructed synthetic stereograms based on the right dihedra method applied to random fault-slip data. This simulation experiment was carried out using a modified version of program RDTM.BAS (Ramsay and Lisle, 2000, p. 794). In repeated trials, it was found that the

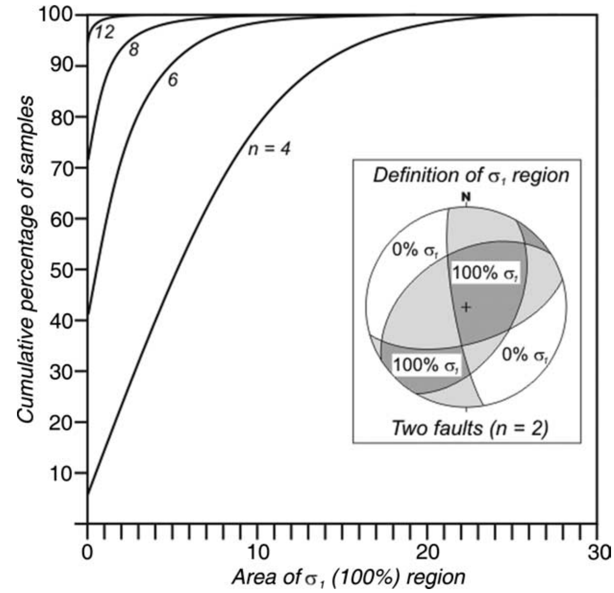

Fig. 1. Right dihedra method. Inset; 100% σ_1 region defined on a stereogram. Graph; percentage of samples of random faults producing a 100% σ_1 region with less than the specified area. The area of the σ_1 area is expressed as a percentage of that of the full stereogram. n is number of faults in the sample. 5000 samples for each sample size were used.

area of the stereogram occupied with σ_1 field that is compatible with 100% of the faults varies as a function of sample size. Fig. 1 shows the cumulative percentage of samples that produce less than the specified area on the stereogram. With large samples of random of faults ($n>12$), it is rare to retain σ_1 directions that are compatible with all of the faults. When $n=8$, 25% of samples produce a σ_1 100% area. However, with $n=5$, 75% of samples give rise to a σ_1 area. This supports the statement by Carey-Gailhardis et al. (1992) that the mere presence of a σ_1 area alone is not a valid criterion for the recognition of homogeneous data.

A further feature of the results is that the area occupied on the stereogram by the compatible σ_1 directions can be large for small samples of faults, but decreases for large samples (Fig. 1 and Table 1). The values in Table 1 can be used as critical values to test the hypothesis that a set of fault-slip data are a random sample from a population of fault-slip pairs with a uniform orientation distribution.

Table 1
Right dihedra method applied to 5000 random samples of faults, for different sample sizes, n. Results are areas of the σ_1 search region as a percentage of the total area of the stereogram. The mean and percentiles of the distribution of areas are given

	Mean σ_1 area	90% percentile	95% percentile	99% percentile
$n=4$	6.29	13.55	16.40	21.13
$n=5$	3.06	7.89	10.41	15.14
$n=6$	1.57	4.73	6.62	10.41
$n=7$	0.82	2.52	4.42	8.20
$n=8$	0.40	1.26	2.52	5.36
$n=9$	0.21	0.63	1.26	3.79
$n=10$	0.10	0.00	0.63	2.21
$n=12$	0.03	0.00	0.00	0.63

In our simulations the average area occupied by compatible σ_1 directions expressed as a percentage of the total area of the stereogram (Table 1), approximates to

$$\text{Mean area }(\%) = 100(0.5^n) \quad (1)$$

For example, with samples of five random faults, the average size of the 100% σ_1 area is about 3%. This demonstrates that the right dihedra method, when applied to small samples of faults, frequently produces σ_1 search areas of small dimensions. With real data, it would be therefore unjustified to conclude that a tightly constrained σ_1 area is an indication of good data quality.

4. Grid search method

Program SLICK.BAS (Ramsay and Lisle, 2000, pp. 805–807) is a typical grid search method of stress inversion. This method searches for stress tensors capable of producing directions of shear stress on the observed faults that, within some acceptable limits, are parallel to the observed slip lineations. To simplify the discussion, in this paper we will refer to the angular discrepancy between the actual and predicted slip lineations as the pitch misfit. Program SLICK.BAS takes no account of the sense of slip on the faults. During the search, a full variety of trial tensors are considered by systematically varying the three variables that specify the orientation of the stress tensor and a fourth variable, Φ, which specifies the shape of the stress tensor, i.e.

$$\Phi = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} \quad (2)$$

4.1. Proportion of trial tensors that fit a sample of random faults

Intuitively, it would seem unlikely that stress tensors could be found that would match a sample of random fault-slip data. Can the quality of natural fault-slip data therefore be assessed by examining the proportion of trial tensors that match the data? Some pertinent results for random data can be anticipated from the laws of probability. For a single random fault-slip direction data pair, a certain proportion of the considered tensors produce theoretical slip directions close to the actual direction present on the fault. The average of this proportion, P, will be equal to p, the tolerance angle of acceptable lineation pitches (e.g. 10°) expressed as a proportion of the maximum range of absolute value of pitch (90° in this study since slip sense is not considered). For n random faults it can be shown (Appendix A) that the mean proportion of tensors producing average pitch misfits of less than p is equal to

$$P = \frac{(np)^n}{n!} \quad (3)$$

Proportions calculated from Eq. (3) are shown in Table 2, which can be interpreted as the probability (as a percentage) that a given stress tensor will match the random fault-slip data. However, the average number of fitting tensors found by grid search is equal to the tabulated proportion P multiplied by the number of tensors examined. The latter is governed by the increments of the variables used in the 4D grid search, and in our simulations this number was typically over 300,000. The information in Table 2 is difficult to use as the basis of a test of data quality because it relates to average proportions, rather than to the results from an individual sample. However, the present results serve to emphasise the fact that the existence of fitting tensors does not prove that a given data set meets the requirements for valid palaeostress determination.

Table 2
Percentage of trial stress tensors compatible with samples of random striated faults. A tensor is compatible when the mean absolute value of the pitch misfits (deviations between actual and predicted slip direction) is less than a specified threshold

n (number of random faults in the sample)	Allowable angle of pitch misfit (°)	p=5/90 (10/90)	P=% trial tensors that explain fault-slip data
1	±5 (±10)	0.055 (0.11)	5.55 (11.10)
2	±5 (±10)	0.055 (0.11)	0.62 (2.47)
3	±5 (±10)	0.055 (0.11)	0.07 (0.62)
5	±5 (±10)	0.055 (0.11)	0.0014 (0.041)
10	±5 (±10)	0.055 (0.11)	7.7E^{-8} (7.9E^{-5})

4.2. The average angular difference between predicted and actual striations

Although some inversion methods use other measures of misfit between observed and theoretical geometry of fault-slip (e.g. Gephart, 1990), the majority of results of palaeostress analysis are qualified by quoting the average pitch misfit for all faults in the sample. A low mean pitch misfit is taken as indicative of a good solution. Commonly, published palaeostress analysis yield tensors that produce an average pitch misfit as high as 10°. This equates to a value of 20° of ANG, the quality estimator used by Angelier (1994), which takes the slip sense into account and therefore has a range of 0–180°. Experiments using program SLICK.BAS allow the distribution of the average pitch misfits to be determined for differently sized samples of random fault-lineation data (Fig. 2).

The results in Fig. 2 reveal that the average misfit increases with the sample size and approaches a value of 45° for very large samples. For samples containing 15 or more faults, 95% of samples produce an average misfit of greater than 17° (equivalent to an ANG value of 34°), a figure greater than those commonly reported in published palaeostress analyses. The average misfit is therefore a useful quality indicator when the sample size is large. However, for small samples ($n<10$), the 5 and 95% values of the distribution of average misfit enclose the typical values of published results. For example there are many cases where tensors are calculated with as few as five faults. In such cases, our results show that mean misfits of less than 2° are required to allow rejection of the hypothesis that the sample consists of random faults with random lineations. When sources of measurement errors are also considered, it becomes difficult to draw conclusions based on mean pitch misfit values.

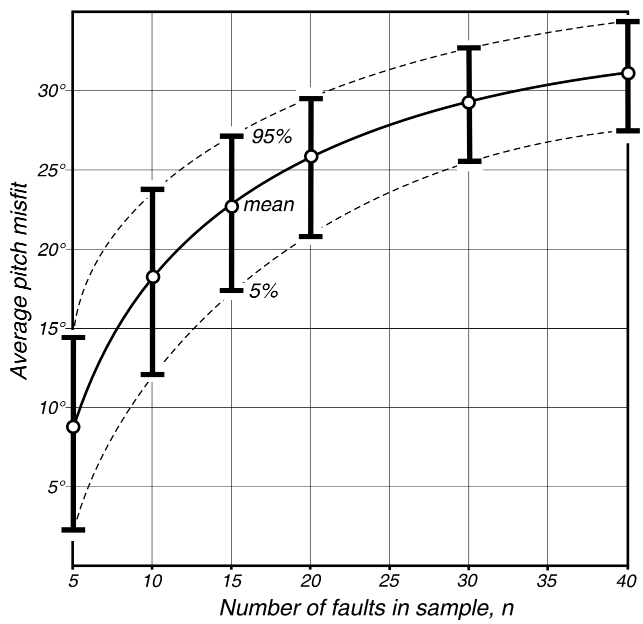

Fig. 2. Average pitch misfit (angle in degrees) for all faults in a sample as a function of sample size. The mean, the 5% and the 95% points of the distribution of average misfit are shown.

Therefore with small samples ($n<10$) this quality indicator appears to not be very useful.

5. Conclusions

The application of stress inversion methods to fault-slip data is fraught with dangers. The assumption that the faults recorded in the field are homogeneous in terms of the bulk stress that brought about their displacements may be valid in certain instances, but this is not a general rule and therefore the onus is on the structural geologist to test the validity of this assumption in each particular case. Where large numbers of data have been assembled from stations, there are possible checks on the homogeneity assumption. For instance, methods can be used that relax the assumption and allow for the data consisting of a small number of homogeneous subgroups (Nemcok and Lisle, 1995; Yamaji, 2003). Alternatively, if sample sizes are sufficiently large there are other tests of the homogeneity of the data based on the variability of the stress tensors obtained from analysis of smaller sub-samples, e.g. the bootstrap-type approach (Arlegui-Crespo and Simón-Gómez, 1998; Albarello, 2000). For small samples, certain indicators have been traditionally used to assess the quality of the data. The present study demonstrates that several of these indicators are unreliable in the case of small samples:

(1) *The existence of σ_1 orientations compatible with 100% of the fault data, as constructed using the right dihedra method.* This is not reliable because a 100% region is found to be a common feature of stereograms produced from small sample sizes. For example, 75% of samples of five meaningless random faults that are unrelated to a stress tensor produce such a region.

(2) *The existence of a region of 100% σ_1 compatibility occupying a small area on the stereogram.* Although this creates the impression that the stress axes are well constrained by the data, we show that small σ_1 areas are frequently produced even from random data. Larger regions of σ_1 are more likely where data are dynamically homogeneous, where a small number of faults are used in the analysis or where a limited range of orientations of faults and their lineations are present in the data set, i.e. where the information content of the sample is small.

(3) *In grid search inversion methods, the average angle of misfit between observed and predicted slip lineations.* This angle is found to be small in the case of random fault data; it is therefore not a valid measure of data quality. This criterion could give a false sense of security in palaeostress work because the misfit angle reduces as the sample size decreases.

The problems of recognising data of poor quality are most acute with small samples of faults. The present work therefore concludes that stress estimates based on less than eight faults should be treated with grave suspicion. It is regrettable that many published analyses have used only this number of striated faults. In the course of any regional study it is advisable to collect a few large datasets (>20 faults), at least at stations where faults occur in sufficient abundance. This will facilitate the judgement of the suitability of the fault-slip data for palaeostress analysis.

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Appendix A. The proportion of stress tensors compatible with the *n* random striated faults

For a single fault plane with randomly selected orientation, the pitches of the theoretical shear stress direction arising from a large number of trial stress tensors, showing an unbiased variation in orientation and stress ratio, will display a uniform orientation distribution. Therefore the probability, p, that the fault's theoretical shear stress direction predicted a particular tensor will be acceptably close in orientation to its actual slip lineation is equal to

$$p=\frac{t}{90}$$

where t is the angular range of acceptable pitches within the theoretical maximum range of 90°. For example, a tolerance of $\pm 10°$ gives $p=1/9$.

The derivation of the probability that a given tensor will match two faults is depicted in Fig. A1a. On a graph of the pitch misfit of fault 1, f_1, against the pitch misfit of fault 2, f_2, a random point represents the predicted misfit arising from a given tensor. Tensors that produce an average misfit of less than t plot below the line $f_1+f_2=2t$. Therefore the number of such tensors expressed as a proportion of all tensors is the area

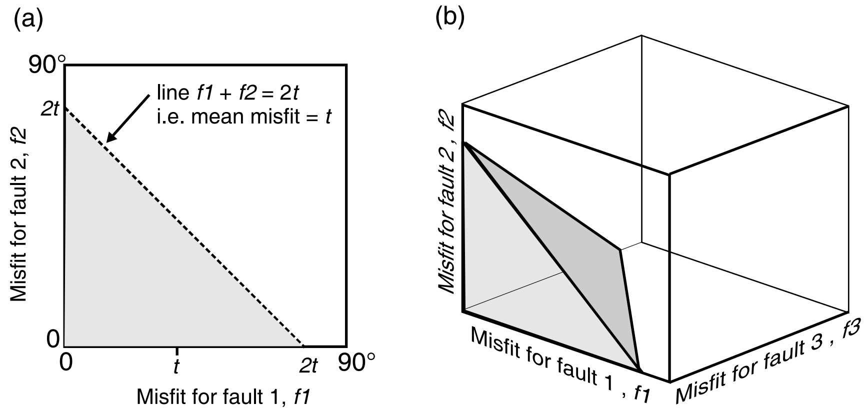

Fig. A1. (a) Two faults with random pitch misfits. Cases where mean misfit $<t$ lie in a shaded triangle and probability of such cases occurring is equal to the area of the shaded triangle as a proportion of the area of the square. (b) For three faults, the proportion of faults with mean misfit $<t$ is equal to the volume of the shaded pyramid as a proportion of the cube.

of the shaded triangle relative to that of the total area on the graph, i.e.

$$P = 0.5\left(\frac{2t}{90}\right)^2 = 0.5(2p)^2$$

By analogy, the proportion of tensors that deliver an average pitch misfit of less than t for samples of three faults equates with the volume of the pyramid in Fig. A1b as a proportion of that of the cube, i.e.

$$P = \frac{(3p)^3}{3!}$$

Generalising for any sample size n, the proportion of tensors that produce an average pitch misfit of less than t is the volume of a hyper-pyramid as a fraction of the volume of the hyper-cube, i.e.

$$P = \frac{(np)^n}{n!}$$

This is the derivation of Eq. (3).

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